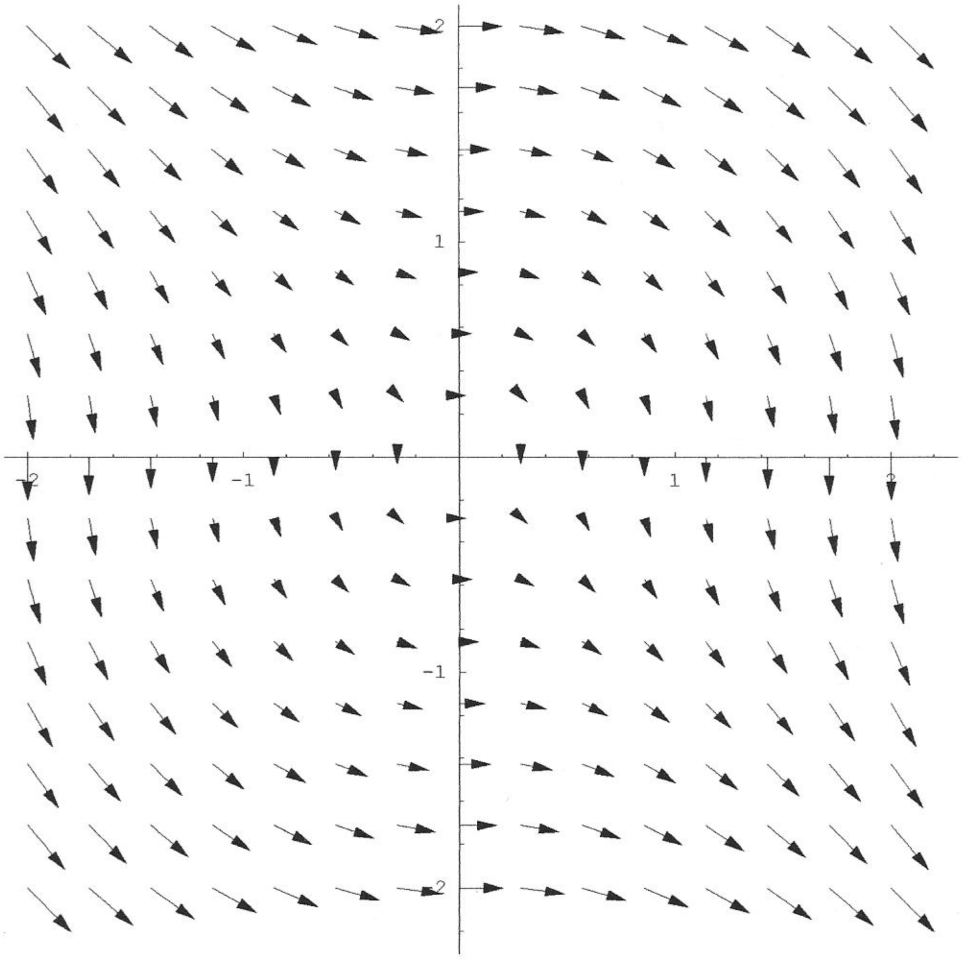
Ex. 1



$\vec{E} = |y|\vec{e}_x - |x|\vec{e}_y$

Explications :

- $x > 0,\ y > 0 \Rightarrow \vec{E} = y\vec{e}_x - x\vec{e}_y$ (ou avant)

  1). $\vec{E}\cdot\vec{r} = 0 \Rightarrow \vec{E} \perp \vec{r}$

  2). $|\vec{E}| = r$

  Donc $\vec{E}$ pour $x, y > 0$ est orthogonal au vecteur position et son module $= r$ (c'était avec la distance à l'origine)

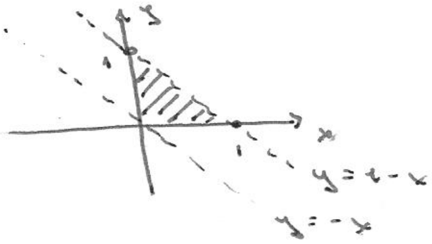
- dans les 3 autres quadrant, on utilise la symétrie. Par exemple, pour $x < 0,\ y > 0$ les composantes E_x, E_y sont les mêmes. (voir dessin).

Ex. 2 | $D = \{(x,y)\in\mathbb{R}^2 : |x|+|y|+|x+y| \leq 2\}$

8 possibilités de choix de signes :

1). $x\geq 0,\ y\geq 0,\ x+y\geq 0 \Rightarrow$

$\Rightarrow 2x+2y\leq 2 \Rightarrow y\leq 1-x$

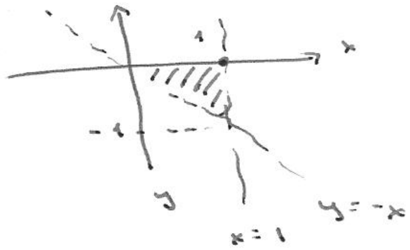


2). $x\geq 0,\ y\geq 0,\ x+y\leq 0 \Rightarrow$

$\Rightarrow 0\leq 2$ l'inégalité vérifiée, mais la seule solution c'est $x=0,\ y=0$

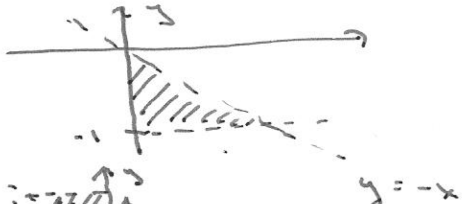
3). $x\geq 0,\ y\leq 0,\ x+y\geq 0 \Rightarrow$

$\Rightarrow x-y+x+y\leq 2 \Rightarrow x\leq 1$



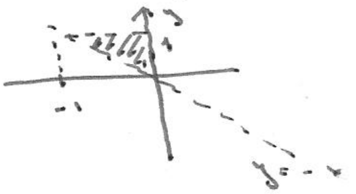
4). $x\geq 0,\ y\leq 0,\ x+y\leq 0 \Rightarrow$

$\Rightarrow x-y-x-y\leq 2 \Rightarrow y\geq -1$



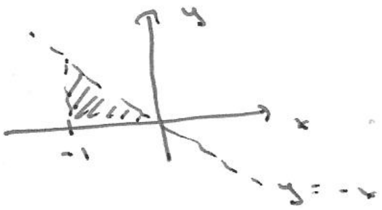
5). $x\leq 0,\ y\geq 0,\ x+y\geq 0 \Rightarrow$

$\Rightarrow -x+y+x+y\leq 2 \Rightarrow y\leq 1$



6). $x\leq 0,\ y\geq 0,\ x+y\leq 0 \Rightarrow$

$\Rightarrow -x+y-x-y\leq 2 \Rightarrow x\geq -1$

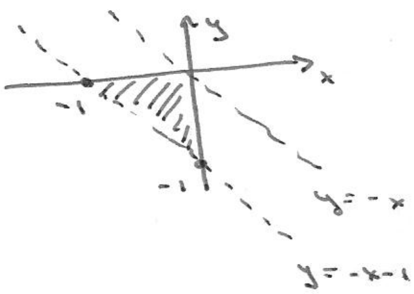


7). $x\leq 0,\ y\leq 0,\ x+y\geq 0 \Rightarrow$

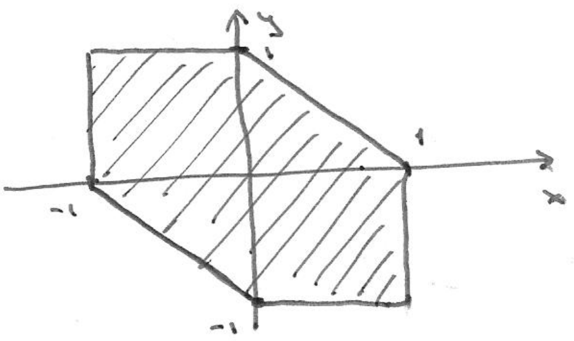
$\Rightarrow -x-y+x+y\leq 2 \Rightarrow$ point $(0,0)$.

8). $x\leq 0,\ y\leq 0,\ x+y\leq 0 \Rightarrow$

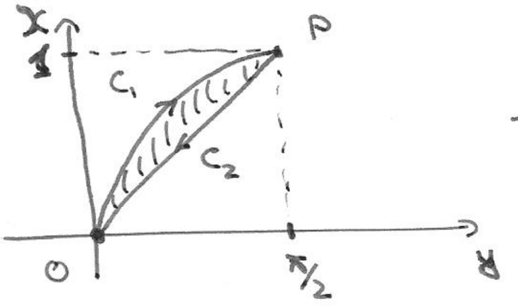
$\Rightarrow -x-y-x-y\leq 2 \Rightarrow y\geq -x-1$



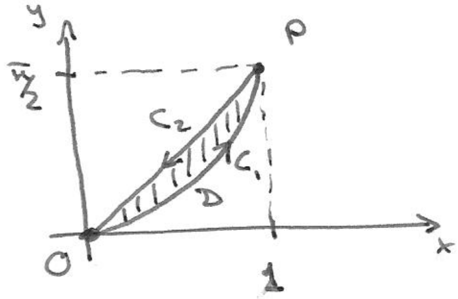
Donc D est le suivant:



Ex. 31



$\Longrightarrow$



Paramétrisation de C_1 :

$$\begin{cases} y = t \\ x = \sin t \\ t \in [0, \pi/2] \end{cases} \Rightarrow \int_{C_1} \sin y \, dx + x \, dy = \int_0^{\pi/2} \sin t \, d(\sin t) + \sin t \, dt =$$

$$= \left[\frac{\sin^2 t}{2} - \cos t \right]_0^{\pi/2} = \frac{1}{2} + 1 = \frac{3}{2}$$

Paramétrisation de C_2 :

$$\begin{cases} x = 0 \cdot t + 1 \cdot (1-t) = 1 - t \\ y = 0\, t + \frac{\pi}{2}(1-t) = \frac{\pi}{2}(1-t) \\ t \in [0,1] \end{cases} \Rightarrow \begin{matrix} dx = -dt \\ dy = -\frac{\pi}{2} dt \end{matrix}$$

$$\int_{C_2} \sin y \, dx + x \, dy = \int_0^1 \sin \frac{\pi}{2}(1-t) \, (-dt) + (1-t)\left(-\frac{\pi}{2}\right) dt =$$

$$= -\int_0^1 \cos \frac{\pi t}{2} \, dt + \frac{\pi}{2} \int_0^1 (t-1) \, dt = - \frac{\sin \frac{\pi t}{2}}{\pi/2} \Big|_0^1 + \frac{\pi}{2} \frac{(t-1)^2}{2} \Big|_0^1 =$$

$$= -\frac{2}{\pi} - \frac{\pi}{4}.$$

Donc: $\displaystyle \int_{C_1} + \int_{C_2} = \frac{3}{2} - \frac{2}{\pi} - \frac{\pi}{4}.$

Thm de Green)

$$\int_{C_1} + \int_{C_2} = \iint_D \left(\frac{\partial}{\partial x}(x) - \frac{\partial \sin y}{\partial y} \right) dx\,dy = \iint_D (1 - \cos y) \, dx\,dy =$$

$$= \int_0^{\pi/2} \left(\int_{\frac{2}{\pi} y}^{\sin y} (1 - \cos y) \, dx \right) dy = \int_0^{\pi/2} (1 - \cos y)\left(\sin y - \frac{2}{\pi} y\right) dy =$$

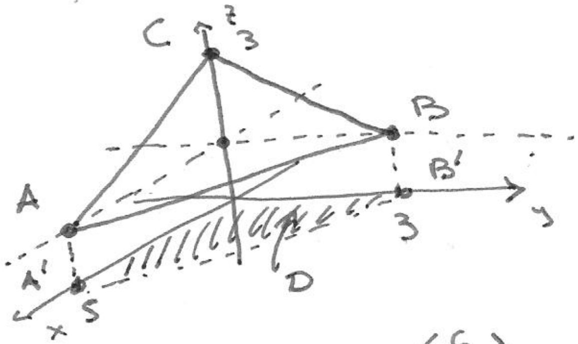
$$= \int_0^{\pi/2} \left(\sin y - \frac{\sin 2y}{2} - \frac{2}{\pi} y + \frac{2}{\pi} y \cos y \right) dy =$$

$$= \left[-\cos y + \frac{\cos 2y}{4} - \frac{y^2}{\pi} \right]_0^{\pi/2} + \frac{2}{\pi} \int_0^{\pi/2} y \, d(\sin y) =$$

$$= -\frac{\pi}{4} + 1 - \frac{1}{2} + \frac{2}{\pi} \left[y \sin y + \cos y \right]_0^{\pi/2} = -\frac{\pi}{4} + \frac{1}{2} + \frac{2}{\pi}\left[\frac{\pi}{2} - 1\right] =$$

$$= -\frac{\pi}{4} - \frac{2}{\pi} + \frac{3}{2}.$$

Ex. 4 $A = (5,0,1)$, $B=(0,3,1)$, $C=(0,0,3)$.



$$\vec{CA} = (+5, 0, -2)$$
$$\vec{CB} = (0, 3, -2)$$

$$\vec{CA} \wedge \vec{CB} = \begin{pmatrix} 5 \\ 0 \\ -2 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 3 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}$$

Donc $\vec{n} = \alpha \begin{pmatrix} 6 \\ 10 \\ 15 \end{pmatrix}$; $|\vec{n}| = 1 \iff \alpha^2 (36 + 100 + 225) = 1$

$361\,\alpha^2 = 1 \Rightarrow \alpha = 19.$

et finalement $\vec{n} = \begin{pmatrix} 6/19 \\ 10/19 \\ 15/19 \end{pmatrix}.$

(a)

Équation du plan ABC:

$6x + 10y + 15z + \beta = 0$ avec β inconnu

mais comme $C \in$ ce plan, on a

$6 \cdot 0 + 10 \cdot 0 + 15 \cdot 3 + \beta = 0 \Rightarrow \beta = -45$

d'où:

(b) $6x + 10y + 15z - 45 = 0.$

Pour calculer le flux:

paramétrisation
$$\begin{cases} x = u \\ y = v \\ z = \dfrac{45 - 6u - 10v}{15} = 3 - \dfrac{2}{5}u - \dfrac{2}{3}v \\ (u, v) \in D \end{cases} \Rightarrow \quad \vec{r}'_u = \begin{pmatrix} 1 \\ 0 \\ -\frac{2}{5} \end{pmatrix}, \vec{r}'_v = \begin{pmatrix} 0 \\ 1 \\ -\frac{2}{3} \end{pmatrix}$$

$$\vec{r}'_u \wedge \vec{r}'_v = \begin{pmatrix} 2/5 \\ 2/3 \\ 1 \end{pmatrix}$$

Donc

$$\iint_{ABC} \vec{E} \cdot d\vec{S} = \iint_D \vec{E} \cdot (\vec{r}'_u \wedge \vec{r}'_v)\, du\, dv =$$

équation de la droite A'B'

$$= \iint_D \left(\frac{2}{5}u^2 + v^2 \right) du\, dv = \int_0^5 \left(\int_0^{3 - \frac{3}{5}u} \left(\frac{2}{5}u^2 + v^2 \right) dv \right) du$$

$$= \int_0^5 \left(\frac{2}{5}u^2 \left(3 - \frac{3}{5}u\right) - \frac{9}{125}(u-5)^3 \right) du = \left[\frac{2}{5}u^3 - \frac{3}{50}u^4 - \frac{9}{500}(u-5)^4 \right]_0^5 =$$

$$= 2 \cdot 25 - \frac{3}{2} \cdot 25 + \frac{9 \cdot 5}{4} = \frac{95}{4}.$$